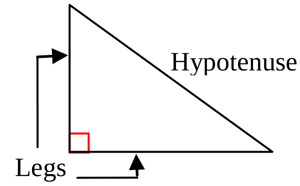
Lesson 4-6: Congruence in Right Triangles

Another very common and useful triangle

The right triangle is an other very common triangle in the real world. If you think about it, for every isosceles triangle, there are two right triangles. Make sense? Sure! Just divide the isosceles triangle in half…two right triangles, back-to-back.

The parts of a right triangle

The side opposite the right angle is called the *hypotenuse* and it is the longest side. The other sides are the *legs*.

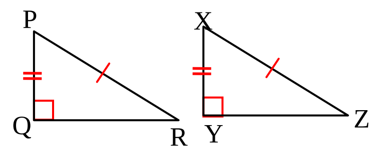


SSA doesn't work…except…

As we've discussed before, you can not rely on SSA to determine triangle congruence. There is however, a "special case" with right triangles. It is called the Hypotenuse-Leg (HL) Theorem. But be careful! It only works consistently with right triangles!

Theorem 4-6 Hypotenuse-Leg (HL) Theorem

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent. $\Delta PQR \cong \Delta XYZ$



Proof

Given: ΔPQR and ΔXYZ are right triangles with right angles Q and Y respectively.

$\overline{PR} \cong \overline{XZ}$ and $\overline{PQ} \cong \overline{XY}$

Prove: $\Delta PQR \cong \Delta XYZ$

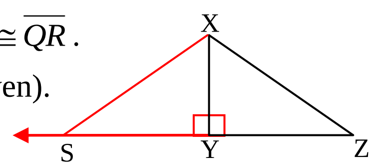
OK, we need a plan here. What if we flip ΔPQR around and put it against ΔXYZ to form an isosceles triangle? Wouldn't that allow us to use the isosceles triangle theorem and conclude that $\angle R$ and $\angle Z$ are congruent? Let's work with it…

Proof: From ΔXYZ draw $\overrightarrow{ZY}$. On $\overrightarrow{ZY}$ mark point S so that $\overline{YS} \cong \overline{QR}$.

We have $\angle PQR \cong \angle XYS$ (rt. $\angle$'s) and $\overline{PQ} \cong \overline{XY}$ (given).

So $\Delta PQR \cong \Delta XYS$ by SAS,

and $\overline{PR} \cong \overline{XS}$ by CPCTC.



We have now flipped ΔPQR around and put it against ΔXYZ and called it ΔXYS.

Now, looking at ΔXYS and ΔXYZ …

$\overline{XS} \cong \overline{XZ}$ by the transitive POC ($\overline{XS} \cong \overline{PR}$, $\overline{PR} \cong \overline{XZ}$).

This makes ΔSXZ an isosceles triangle (two sides are congruent).

$\angle S \cong \angle Z$ by the Isosceles Triangle Theorem.

So $\Delta XYS \cong \Delta XYZ$ by AAS ($\angle S \cong \angle Z$, $\angle XYS \cong \angle XYZ$, $\overline{SY} \cong \overline{YZ}$)

Therefore $\Delta PQR \cong \Delta XYZ$ by transitive POC ($\Delta PQR \cong \Delta XYS$, $\Delta XYS \cong \Delta XYZ$)

Q.E.D.

Lesson 4-6: Congruence in Right Triangles

Using the HL Theorem

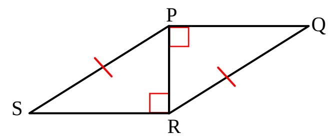
It's simple…to use the HL Theorem you just need to show three things:

1. The two triangles are right triangles.
2. Their hypotenuses are congruent.
3. They have one pair of congruent legs.

Show these three things in any order and you can conclude the HL Theorem applies and that the triangles are congruent.

Examples – Pg 219 & 220

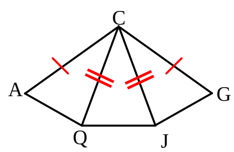
2. Explain why the two triangles are congruent.



- Hypotenuse: $\overline{SP} \cong \overline{QR}$ (Given)
- Right triangle: $\angle SRP \cong \angle QPR$ (All rt. $\angle$'s are $\cong$)
- One leg: $\overline{PR} \cong \overline{RP}$ (Reflexive POC)

Therefore $\Delta SPR \cong \Delta QRP$ (HL)

8. What additional information do you need to prove ΔACQ and ΔGCJ congruent by HL?

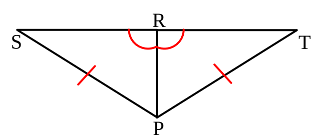


Need right angles, i.e. either:

- $\angle A$ and $\angle G$, or
- $\angle AQC$ and $\angle GJC$

are right angle pairs. Will then also have a congruent hypotenuse and leg.

12. Complete the proof.



Given: $\overline{PS} \cong \overline{PT}, \angle PRS \cong \angle PRT$

Prove: $\Delta PRS \cong \Delta PRT$

Proof: $\angle PRS \cong \angle PRT$ and are suppl $\angle$'s	Given (by diagram)
$\angle PRS$ & $\angle PRT$ are rt. $\angle$'s	a. $\cong$ Suppl. $\angle$'s are rt. $\angle$'s
ΔPRS & ΔPRT are rt. Δ's	b. Defn. of rt. Δ
$\overline{PS} \cong \overline{PT}$	c. Given
$\overline{PR} \cong \overline{PR}$	d. Reflexive POC
$\Delta PRS \cong \Delta PRT$	e. HL

Assign homework

p. 219 #1-11 odd, 14-17, 19, 20, 28, 31, 35-46